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ISE 5643 — Spring 2015 **HW 7** Due: Mar. 30

Problem 5.6

Consider the minimization of $f(x)$ over a feasible region denoted by S. Suppose $\bigtriangledown f(x) \neq 0$ for every $x \in S$. What can you say about the nature of optimal solution to this problem and why?

Ans: The nature of optimal solution to this problem is to change the optimization as a constrained problem. The feasible region will be reduced considering some constraints, which may lead to $\bigtriangledown f(x) \neq 0$ within the feasible region. Finally, we can use some methods to converts this constrained optimization to unconstrained optimization problem.

Problem 5.7

Use the Lagrange multiplier method to solve

$$\begin{aligned} \text{Minimize} \quad & z = x_1^2 + x_2^2 + x_3^2 \\ \text{Subject to} \quad & x_1 + 2x_2 + 3x_3 = 7 \\ & 2x_1 + 2x_2 + x_3 = \frac{9}{2} \end{aligned}$$

Explain (in two or three lines) why your procedure guarantees an optimum solution.

Ans:

(1) First, Constructing the Lagrange function:

$$L(x; v) = x_1^2 + x_2^2 + x_3^2 - v_1(x_1 + 2x_2 + 3x_3 - 7) - v_2(2x_1 + 2x_2 + x_3 - \frac{9}{2})$$

(2) Setting $\partial L / \partial x_r = 0$ for each $r = 1, 2, 3$ gives

$$x_1 = \frac{v_1 + 2v_2}{2}, x_2 = v_1 + v_2, x_3 = \frac{3v_1 + v_2}{2}$$

(3) Further, Substituting the above equations to the constraint condition.

$$\begin{aligned} v_1\left(\frac{v_1 + 2v_2}{2} + 2(v_1 + v_2) + 3(\frac{3v_1 + v_2}{2}) - 7\right) &= 0 \\ v_2\left(2(\frac{v_1 + 2v_2}{2}) + 2(v_1 + v_2) + \frac{3v_1 + v_2}{2} - \frac{9}{2}\right) &= 0 \end{aligned}$$

(4) we get that $v_1 = 0, v_2 = 0; v_1 = 1, v_2 = 0; v_1 = 0, v_2 = 1;$

- For $v_1 = 0, v_2 = 0$, we get $x_1 = 0, x_2 = 0, x_3 = 0$; discard this point.
- For $v_1 = 1, v_2 = 0$, we get $x_1 = 0.5, x_2 = 2, x_3 = 1.5$;

– For $v_1 = 0, v_2 = 1$, we get $x_1 = 1, x_2 = 2, x_3 = 0.5$.

Because the $H_z = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ is PD matrix, so the stationary point $(0.5, 2, 1.5), (1, 2, 0.5)$ are global optimum solution.

Problem 5.9

Find the shortest distance from the point (1,0) to the parabola $y^2 = 4x$ by:

(a) Eliminating the variable y

(b) The Lagrange multiplier technique

Explain why procedure (a) fails to solve the problem, while procedure (b) does not fail.

Ans: The square of the distance is

$$\begin{aligned} \min d(x, y) &= (x-1)^2 + y^2 \\ where\ y^2 &= 4x \end{aligned}$$

Using the following two method to solve it:

(a) $d(x, y) = (x-1)^2 + 4x = (x+1)^2$, it's obvious that when $x = -1$, the function get a minimum value. but $x = -1$ is contradict with $y^2 = 4x$ which means x is a non-negative value.

(b) Constructing the Lagrange function:

$$L(x, y; v) = (x-1)^2 + y^2 - v(y^2 - 4x)$$

First, Setting $\partial L/\partial x = 0 \Longrightarrow x = 1 - 2v; \partial L/\partial y = 0 \Longrightarrow y = 0\ or\ v = 1$, Constructing the $H_L = \begin{pmatrix} 2 & 2 \\ 0 & 2(1-v) \end{pmatrix}$, For the optimal point to be a local minimum H_L must be PD. $\Longrightarrow 2(1-v) > 0 \Longleftrightarrow v < 1$. For $0 = 4(1-2v) \Longrightarrow v = \frac{1}{2} \Longrightarrow x = 0$; So we get the optimal point is $(0, 0)$, the distance is 1.

The reason why method (a) fail to solve the problem is it leave out the constrain condition that x is non-negative when eliminating variable y.