

Problem 1

Given the function to be minimized:

$$g(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

and the starting point $x^{(0)} = [-1.2, 0]^T$

1. Perform four iterations of Nelder and Mead simplex search method by hand using the other two points of the simplex as $[-0.8, -0.4]^T$ and $[-0.8, 0.4]^T$. Use $\alpha = 2$, $\beta = 0.5$, $\gamma = 2$

- (1). Sort: $f(-1.2, 0) = 212.2$, $f(-0.8, -0.4) = 111.4$, $f(-0.8, 0.4) = 9$,
 $x_m = (-0.8, 0.4)$, $x_M = (-1.2, 0)$, $\bar{x} = (-0.8, 0)$.
 Reflection: $x_r = \bar{x} + \alpha(\bar{x} - x_M) = (0, 0)$, $f(x_r) = 1 < f(x_m) = 9$,
 Expansion: $x_e = \bar{x} + \gamma(x_r - \bar{x}) = (0.8, 0)$, $f(x_e) = 41 > f(x_r) = 1$, so $x_M = (0, 0)$
- (2). Sort: $f(-0.8, -0.4) = 111.4$, $f(-0.8, 0.4) = 9$, $f(0, 0) = 1$,
 $x_m = (0, 0)$, $x_M = (-0.8, -0.4)$, $\bar{x} = (-0.4, 0.2)$.
 Reflection: $x_r = \bar{x} + \alpha(\bar{x} - x_M) = (0.4, 1.4)$, $f(x_r) = 154.12 > f(x_m) = 1$,
 Contraction: $\hat{x} = x_M = (-0.8, -0.4)$, $x_c = \bar{x} + \beta(\hat{x} - \bar{x}) = (-0.6, -0.1)$, $f(\hat{x}) = 111.4 > f(x_c) = 23.72$, So $x_M = (-0.6, -0.1)$
- (3). Sort: $f(-0.6, -0.1) = 23.72$, $f(-0.8, 0.4) = 9$, $f(0, 0) = 1$,
 $x_m = (0, 0)$, $x_M = (-0.6, -0.1)$, $\bar{x} = (-0.4, 0.2)$.
 Reflection: $x_r = \bar{x} + \alpha(\bar{x} - x_M) = (0, 0.8)$, $f(x_r) = 65 > f(x_m) = 1$,
 Contraction: $\hat{x} = x_M = (-0.6, -0.1)$, $x_c = \bar{x} + \beta(\hat{x} - \bar{x}) = (-0.5, 0.05)$, $f(\hat{x}) = 23.72 > f(x_c) = 6.25$, So $x_M = (-0.5, 0.05)$
- (4). Sort: $f(-0.8, 0.4) = 9$, $f(-0.5, 0.05) = 6.25$, $f(0, 0) = 1$,
 $x_m = (0, 0)$, $x_M = (-0.8, 0.4)$, $\bar{x} = (-0.25, 0.025)$.
 Reflection: $x_r = \bar{x} + \alpha(\bar{x} - x_M) = (0.85, -0.725)$, $f(x_r) = 209.5481 > f(x_m) = 1$,
 Contraction: $\hat{x} = x_M = (-0.8, 0.4)$, $x_c = \bar{x} + \beta(\hat{x} - \bar{x}) = (0.525, 0.2125)$, $f(\hat{x}) = 9 > f(x_c) = 0.6241$, So $x_M = (0.525, 0.2125)$
 Return:(0.0083, 0.0875)

2. Perform two pattern moves with Hooke and Jeeves method by hand using $\Delta x_1 = 0.1$ and $\Delta x_2 = 0.2$ and $\alpha = 2$.

- (1). $g(x^0) = 212.2$, exploration around x_0 . $x_1 = -1.2 + 0.1 = -1.1$,
 $f(-1.1, 0) = 150.82$, success, $x_2 = 0 + 0.2 = 0.2$, $f(-1.1, 0.2) = 106.42$. so
 Move $x^1 = (-1.1, 0.2) + (-1.1 + 1.2, 0.2 - 0) = (-1, 0.4) = 40 < 106.42$
- (2). $g(x^1) = 40$, exploration around x^1 . $x_1 = -1 + 0.1 = -0.9$,
 $f(-0.9, 0.4) = 20.42 < 40$, success, $x_2 = 0.4 + 0.2 = 0.6$, $f(-0.9, 0.6) = 8.02$.
 so Move $x^2 = (-0.9, 0.6) + (-0.9 + 1, 0.6 - 0.4) = (-0.8, 0.8) = 5.8 < 8.02$