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ISE 5643 — Spring 2015 **HW 4** Due: Feb. 25

Problem 1

Find and classify the stationary points of

$$f(x) = 2x_1^3 + 4x_1x_2^2 - 10x_1x_2 + x_2^2$$

as shown in Figure 3.19. **(25 pts)**

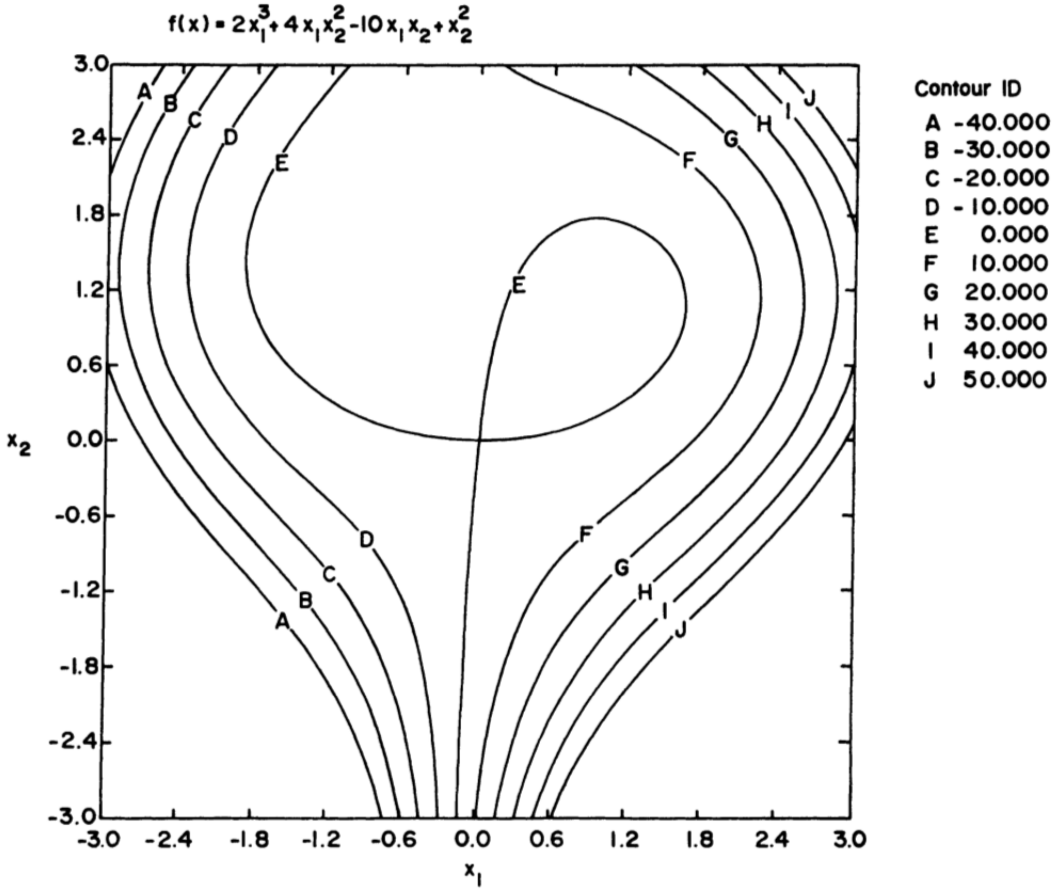

Figure 3.19. Function of problem 3.11.

Solving the following equation to find the stationary points:

$$\begin{aligned}\frac{\partial f}{\partial x_1} &= 6x_1^2 + 4x_2^2 - 10x_2 \\ \frac{\partial f}{\partial x_2} &= 8x_1x_2 - 10x_1 + 2x_2\end{aligned}$$

Therefore, we get three points $(0,0),(1,1),(-\frac{\sqrt{6}}{12}-\frac{3}{4},2-\frac{\sqrt{6}}{8})$ Applying the following equation to classify these points:

$$\begin{aligned}
A &= \frac{\partial^2 f}{\partial x_1^2} = 12x_1 \\
B &= \frac{\partial^2 f}{\partial x_2^2} = 8x_1 + 2 \\
C &= \frac{\partial^2 f}{\partial x_1 x_2} = \frac{\partial^2 f}{\partial x_2 x_1} = 8x_2 - 10 \\
D_2 &= AB - C^2
\end{aligned}$$

(a) For point $(0,0)$, $D=0$, so it's Saddle point;

(b) For point $(1,1)$, $D>0$ *and* $A>0$, so it's local minimum;

(c) For point $(-\frac{\sqrt{6}}{12}-\frac{3}{4},2-\frac{\sqrt{6}}{8})$, $D>0$ *and* $A<0$, so it's local maximum.

Problem 2

Investigate the definiteness of the following quadratic forms: **(15 pts)**

$$\begin{aligned}
Q_1(x) &= x_1^2 + 2x_2^2 - 3x_3^2 - 6x_1x_2 + 8x_1x_3 - 4x_2x_3 \\
Q_2(x) &= 2ax_1x_2 + 2bx_2x_3 + 2cx_3x_1 \\
Q_3(x) &= x_1^2 + 5x_2^2 + 3x_3^2 + 4x_1x_2 - 2x_2x_3 - 2x_1x_3
\end{aligned}$$

(a) For $Q_1(x)$

$$A = \begin{pmatrix} 1 & -3 & 4 \\ -3 & 2 & -2 \\ 4 & -2 & -3 \end{pmatrix}$$

$|D_1| = 1 > 0$,$|D_2| = -7 < 0$,$|D_3| = 33 > 0$, so $Q_1(x)$ is indefinite.

(b) For $Q_2(x)$:

$$A = \begin{pmatrix} 0 & a & c \\ a & 0 & b \\ c & b & 0 \end{pmatrix}$$

$|D_1| = 0$,$|D_2| = -a^2 < 0$,$|D_3| = 2abc$, so $Q_2(x)$ is indefinite.

(c) For $Q_3(x)$:

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

$|D_1| = 1 > 0$,$|D_2| = 1 > 0$,$|D_3| = 1 > 0$, so $Q_3(x)$ is positive definite.

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Problem 3

Suppose at $x = \bar{x}$, $\nabla f(\bar{x}) = 0$. What can you say about $\bar{x}$ if **(15 pts)**

(a) $f(x)$ is convex?
$\bar{x}$ is a global minimum point.

(b) $f(x)$ is concave?
$\bar{x}$ is global maximum point.

(c) $\nabla^2 f(\bar{x})$ is indefinite?
$\bar{x}$ is Saddle point.

(d) $\nabla^2 f(\bar{x})$ is positive semidefinite?
$\bar{x}$ is a local minimum point.

(e) $\nabla^2 f(\bar{x})$ is negative semidefinite?
$\bar{x}$ is a local maximum point.

Problem 4

While searching for the minimum of **(15 pts)**

$$f(x) = [x_1^2 + (x_2+1)^2][x_1^2 + (x_2-1)^2]$$

we terminate at the following points:

(a) $x^{(1)} = [0,0]^T$

(b) $x^{(2)} = [0,1]^T$

(c) $x^{(3)} = [0,-1]^T$

(d) $x^{(4)} = [1,1]^T$

Classify each point.
Applying the following equation:

$$\begin{aligned} A &= \frac{\partial^2 f}{\partial x_1^2} = 12x_1^2 + 4x_2^2 + 4 \\ B &= \frac{\partial^2 f}{\partial x_2^2} = 12x_2^2 + 4x_1^2 - 4 \\ C &= \frac{\partial^2 f}{\partial x_1 x_2} = \frac{\partial^2 f}{\partial x_2 x_1} = 8x_1 x_2 \\ D_2 &= AB - C^2 \end{aligned}$$

(a) For point $x^{(1)}$, $D_1 = 4 > 0, D_2 = -16 < 0$, so it's Saddle point;

(b) For point $x^{(2)}$, $D_1 = 8 > 0, D_2 = 64 > 0$, so it's minimum points ;

(c) For point $x^{(3)}$, $D_1 = 8 > 0, D_2 = 64 > 0$, so it's minimum points

(d) For point $x^{(4)}$, $D_1 = 16 > 0, D_2 = 176 > 0$, so it's minimum points;

Problem 5

Find the dimensions of the minimum-cost open-top rectangular container.(Let $v^* = 10m^3$.) **(15 pts)**

Let $l > 0, w > 0, h > 0$,denoted as the length,width, and height of this container, so the objective function $C = 2lh + 2hw + lw$ with $lwh = 10$. By substituting $w = \frac{10}{lh}$, we get:

$$\begin{aligned} C &= 2lh + \frac{20}{l} + \frac{10}{h} \\ \frac{\partial C}{\partial l} &= 2h - 20l^{-2} \\ \frac{\partial C}{\partial h} &= 2l - 10h^{-2} \\ H_C &= \begin{pmatrix} 40l^{-3} & 2 \\ 2 & 20h^{-3} \end{pmatrix} \end{aligned}$$

From the first derivative of l, h we can get the stationary point of the objective function, we get $l = w = 2 * (\frac{5}{2})^{\frac{1}{3}}, h = (\frac{5}{2})^{\frac{1}{3}}$. By substituting l, h to the H_C, we get that H_C is positive definite. so the stationary point also is the minimum point.

Problem 6

Test the property of the following matrix(PD,PSD,ND,NSD or indefinite): **(15 pts)**

$$\begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 4 \\ 5 & 8 & 7 \end{bmatrix}$$

$(A + A')/2 =$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 7 \end{bmatrix}$$

$|D_1| = 1 > 0$,$|D_2| = 0$,so this matrix is indefinite.