

**Problem 4.28**

Develop a table similar to Table 4.9 for  $GF(2^4)$  with  $m(x) = x^4 + x + 1$ .

The generator  $g$  must satisfy  $f(g) = g^4 + g + 1 = 0$ . and we will perform modulo 2 on the coefficients. we have the following.

$$\begin{aligned}
 g^4 &= -g - 1 = g + 1 \\
 g^5 &= g(g^4) = g^2 + g \\
 g^6 &= g(g^5) = g^3 + g^2 \\
 g^7 &= g(g^6) = g^4 + g^3 = g + 1 + g^3 \\
 g^8 &= g(g^7) = g^2 + g + g^4 = g^2 + g + g + 1 = g^2 + 1 \\
 g^9 &= g(g^8) = g^3 + g \\
 g^{10} &= g(g^9) = g^4 + g^2 = g + 1 + g^2 \\
 g^{11} &= g(g^{10}) = g^2 + g + g^3 \\
 g^{12} &= g(g^{11}) = g^3 + g^2 + g^4 = g^3 + g^2 + g + 1 \\
 g^{13} &= g(g^{12}) = g^4 + g^3 + g^2 + g = g + 1 + g^3 + g^2 + g = g^3 + g^2 + 1 \\
 g^{14} &= g(g^{13}) = g^4 + g^3 + g = g + 1 + g^3 + g = g^3 + 1
 \end{aligned}$$

$GF(2^4)$  using  $x^4 + x + 1$

Power REP	Polynomial REP	Binary REP	Decimal (Hex) RPEP
0	0	0000	0
$g^0$	1	0001	1
$g^1$	$g$	0010	2
$g^2$	$g^2$	0100	4
$g^3$	$g^3$	1000	8
$g^4$	$g + 1$	0011	3
$g^5$	$g^2 + g$	0110	6
$g^6$	$g^3 + g^2$	1100	12
$g^7$	$g^3 + g + 1$	1011	11
$g^8$	$g^2 + 1$	0101	5
$g^9$	$g^3 + g$	1010	10
$g^{10}$	$g^2 + g + 1$	0111	7
$g^{11}$	$g^3 + g^2 + 1$	1110	14
$g^{12}$	$g^3 + g^2 + g + 1$	1111	15
$g^{13}$	$g^3 + g^2 + 1$	1101	13
$g^{14}$	$g^3 + 1$	1001	9