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ISE 5643 — Spring 2015 **HW 3** Due: Feb. 18

Problem 1

Consider the problem

$$\min f(x) = (x-2)^4 + 2(x-3)^2 + 3 \; over \; x \in [0, 10]$$

Perform five iterations of **(20 pts)**

(1) bisection algorithm

```
interation: 0
interval:(0,10),xm=5,f1(x1)=-44,f1(xm)=116,f1(x2)=2076,
    length=10
interation: 1
interval:(0,5),xm=2.5,f1(x1)=-44,f1(xm)=-1.5,f1(x2)=116,
    length=5
interation: 2
interval:(2.5,5),xm=3.75,f1(x1)=-1.5,f1(xm)=24.4375,f1(x2)
    =116,length=2.5
interation: 3
interval:(2.5,3.75),xm=3.125,f1(x1)=-1.5,f1(xm)=6.19531,f1
    (x2)=24.4375,length=1.25
interation: 4
interval:(2.5,3.125),xm=2.8125,f1(x1)=-1.5,f1(xm)=1.39551,
    f1(x2)=6.19531,length=0.625
interation: 5
interval:(2.5,2.8125),xm=2.65625,f1(x1)=-1.5,f1(xm)
    =-0.244507,f1(x2)=1.39551,length=0.3125
```

(2) secant method

```
L=0,x1=0.207547,R=10,length=-9.79245
interation: 2,
L=0.207547,x1=0.366279,R=10,length=-9.63372
interation: 3,
L=0.366279,x1=0.494379,R=10,length=-9.50562
interation: 4,
L=0.494379,x1=0.60156,R=10,length=-9.39844
interation: 5,
L=0.60156,x1=0.693607,R=10,length=-9.30639
```

(3) Newton-Raphson method

```
interation: 1
x0=10,x1=7.31088,f1(x0)=2076,f1(x1)=616.427,length=2.68912
interation: 2
x0=7.31088,x1=5.51091,f1(x0)=616.427,f1(x1)=183.153,length=1.79997
interation: 3
x0=5.51091,x1=4.30531,f1(x0)=183.153,f1(x1)=54.2271,length=1.2056
interation: 4
x0=4.30531,x1=3.50519,f1(x0)=54.2271,f1(x1)=15.6613,length=0.800123
interation: 5
x0=3.50519,x1=3.00301,f1(x0)=15.6613,f1(x1)=4.04834,length=0.502173
```

Problem 2

(a) By using Golden section search, solve problem 2.23 from textbook (page $66-67$). Use final $|IU| < 0.0001$ for stopping criteria.(you can write a MATLAB code or use Excel). What is the optimal solution? What is the objective function value?**(15 pts)**
Given $L = 1000ft, Q = 20gpm$, our goal is to minimize the objective function

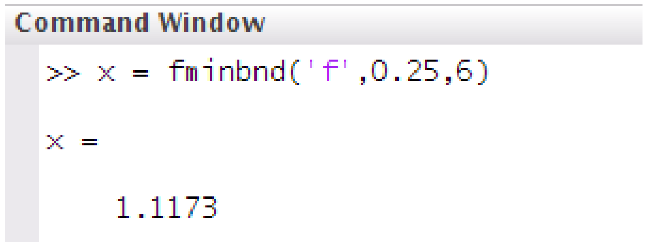
$$\begin{aligned} f &= 0.45 * L + 0.245 * L * D^{1.5} + 325(hp)^{1/2} + 61.6(hp)^{0.925} + 102 \\ where & \\ hp &= 4.4 * 10^{-8}\frac{LQ^3}{D^5} + 1.92 * 10^{-9}\frac{LQ^{2.68}}{D^{4.68}} \end{aligned}$$

to get the optimal pipe diameter $D(in.)$ within $[0.25, 6]$

```
interation: 0, FE=0interval:(0.25,6),x1=2.4463,x2=3.8037,f(x1)=1510.63,f(x2)=2376.47,length=5.75
interation: 1, FE=1interval:(0.25,3.8037),x1=1.60739,x2=2.4463,f(x1)=1113.38,f(x2)=1510.63,length=3.5537
...
interation: 23, FE=23interval:(1.11725,1.11734),x1=1.11728,x2=1.1173,f(x1)=1003,f(x2)=1003,length=8.9733e-05
```

we get that the optimal $D = 1.1173$ and the $f(1.1173) = 1003$.

(b) Solve the problem with an appropriate function in MATLAB Optimization Toolbox. Compare your results with those in (a). **(5 pts)**



The result is the same as my answer.

(c) Why do we need derivative-requiring algorithms (bisection, Newton, secant) instead of solving analytically by taking the derivative and set equal to 0? **(5 pts)** Because this derivative of this objective function is very complex and not easy to computing.

Problem 3

(a) Minimize $f(x) = 3x^4 + (x-1)^2$ over $x \in [0, 3]$

(b) Minimize $f(x) = 2(x-3)^2 + \exp(0.5x^2)$ over $x \in [0, 2.75]$

Solve each of the above functions by using:**(50 pts)**

For function (a)

(1) golden section search

```
interation:  0,  FE=0error:15.012
interval:(0,3),x1=1.1459,x2=1.8541,f(x1)=5.19384,f(x2)
    =36.1827,length=3
interation:  1,  FE=1error:1.79563
interval:(0,1.8541),x1=0.708204,x2=1.1459,f(x1)=0.839811,f
    (x2)=5.19384,length=1.8541
...
interation:  21,  FE=21error:2.83918e-10
interval:(0.450645,0.450768),x1=0.450692,x2=0.450721,f(x1)
    =0.425516,f(x2)=0.425516,length=0.000122569
interation:  22,  FE=22error:1.11398e-09
interval:(0.450645,0.450721),x1=0.450674,x2=0.450692,f(x1)
    =0.425516,f(x2)=0.425516,length=7.57518e-05
```

(2) bisection algorithm

```
interation: 0,error=15.012
interval:(0,3),xm=1.5,f1(x1)=-2,f1(xm)=41.5,f1(x2)=328,length=3
interation: 1,error=0.586202
interval:(0,1.5),xm=0.75,f1(x1)=-2,f1(xm)=4.5625,f1(x2)=41.5,length=1.5
...
interation: 15,error=4.32215e-09
interval:(0.450623,0.450714),xm=0.450668,f1(x1)=-0.000710149,f1(xm)=-0.000283929,f1(x2)=0.00014236,length=9.15527e-05
interation: 16,error=2.62652e-10
interval:(0.450668,0.450714),xm=0.450691,f1(x1)=-0.000283929,f1(xm)=-7.07928e-05,f1(x2)=0.00014236,length=4.57764e-05
```

(3) secant method

```
interation: 1, error=0.538451
L=0,x1=0.0181818,R=3,length=-2.98182
interation: 2, error=0.503927
L=0.0181818,x1=0.0359262,R=3,length=-2.96407
...
interation: 152, error=5.38061e-10
L=0.450687,x1=0.450688,R=3,length=-2.54931
interation: 153, error=4.62094e-10
L=0.450688,x1=0.450689,R=3,length=-2.54931
```

Plot error term versus iteration number with each method for each function. Use the following error term for ith iteration:

$$error_i = abs(f((a_i + b_i)/2) - f(x^*)) \text{ for golden section}$$
$$error_i = abs(f(z_i) - f(x^*)) \text{ for bisection and secant method}$$

where x^* is the optimal solution.

Assuming that initial solution refers to iteration 0, start your plot from iteration 1. Turn in MATLAB or EXCEL solution and the error function plot of (1) – (3) on the same plot for each function.
Use $\epsilon = 0.0001$ as a stopping criterion for bisection and secant method.
Use $|IU| < 0.0001$ as a stopping criterion for golden section search.

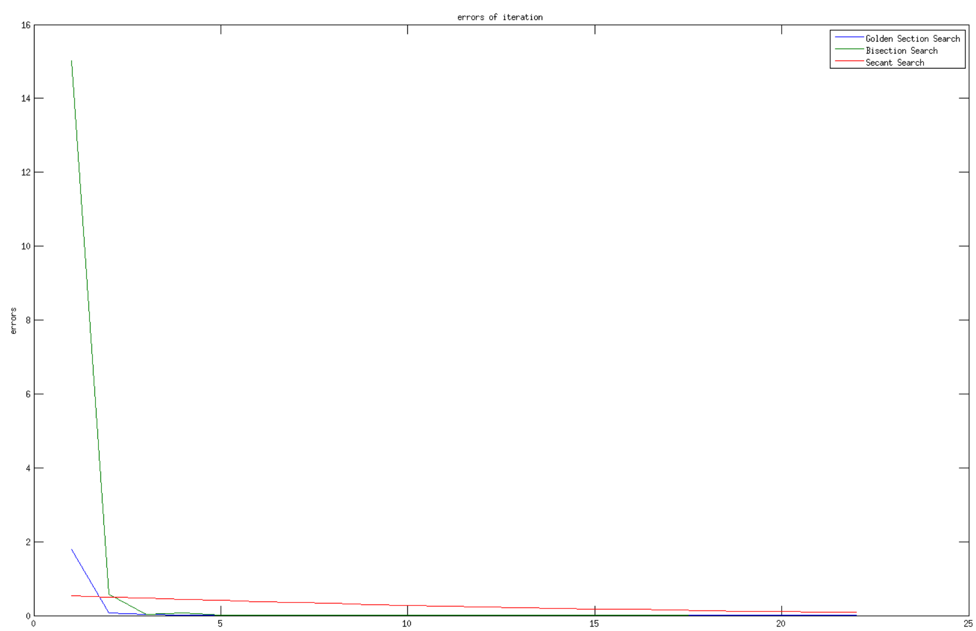
First, using MATLAB tools to get the optimal values $x^* = 0.4507$, then using the above error equations to get the table below[1]. So we can get the plot of function (a).

Function(a)-1

method \ iteration	1	2	3	4	5	6	7
Golden	1.7956	0.0801	0.0388	0.0073	0.0089	0.0002	0.0029
Bisection	15.0120	0.5862	0.0244	0.0662	0.0015	0.0037	0.0001
Secant	0.5385	0.5039	0.4709	0.4392	0.4089	0.3800	0.3524

Function(a)-2

method \ iteration	8	9	10	11	12	13
Golden	0.0001	0.0002	3.043e-06	6.932e-05	4.391e-06	3.043e-06
Bisection	1.8810e-04	1.0425e-06	2.802e-05	4.575e-06	3.131e-07	5.315e-08
Secant	0.3261	0.3012	0.2775	0.2551	0.2340	0.2142



[1]some of the data have been approximated

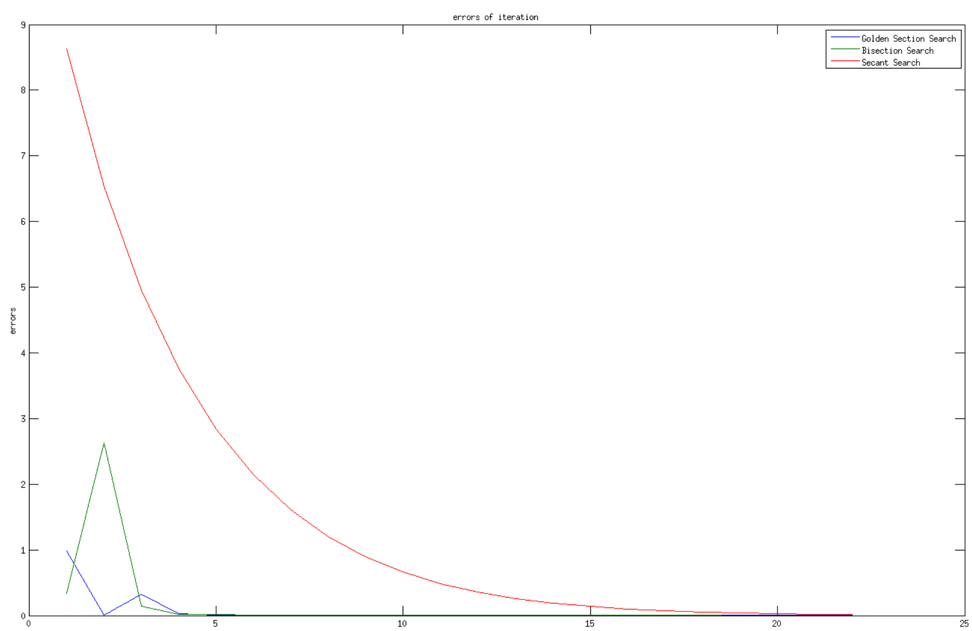
Like above process, the optimal value is $x^* = 1.5907$,we can get the plot of function (b) as below.

Function(a)-1

method \ iteration	1	2	3	4	5	6	7
Golden	0.9855	0.0019	0.3221	0.0325	0.0019	0.0088	0.0001
Bisection	0.3389	2.6312	0.1473	0.0154	0.0150	0.0000	0.0036
Secant	8.6336	6.5324	4.9550	3.7578	2.8439	2.1451	1.6111

Function(a)-2

method \ iteration	8	9	10	11	12	13
Golden	0.0019	1.2704e-04	7.3530e-05	1.3694e-05	1.5069e-05	6.4696e-07
Bisection	0.0008	0.0002	2.7140e-05	1.8173e-06	3.3426e-07	1.4525e-07
Secant	1.2042	0.8956	0.6626	0.4878	0.3573	0.2606



Problem 4

For a unimodal function $f(x)$ we know that $f(1) = 7, f(2) = 3, f(3) = 6$ and $f(4) = 10$. What is the smallest interval in which the minimum of f is located? What will be your response if you are also given the additional information that $f^{'}(2) < 0$?**(5 pts)**

The interval $(1, 3)$ is the smallest interval where the minimum of f. after knowing that $f'(2) < 0$, the smallest interval becomes $(2, 3)$.

Matlab Code

All related code files can be download at my GitHub[2].

Newton Search:

```
% mailto:chao@ou.edu
% Newton's search for minimization of function f(x)
% the same as find the root of f'(x)=0 assuming f'(a)<0 and f'(b)>0;
% f(x)=(x-2)^4+2(x-3)^2+3 -> f'(x)=4(x-2)^3+4(x-3) -> f''(x)=12(x-2)^2+4
epsilon = 0.0001;                                    % accuracy value
k = 0;                                               % number of iterations
iter = 50;                                           % maximun number of iterations

x0=10;
x1=x0-f1(x0)/f2(x0);
f_0=f1(x0);
f_1=f1(x1);
len = x0-x1;
k=k+1;
fprintf(1,'interation: %d\n',k);
fprintf(1,'x0=%g,x1=%g,f1(x0)=%g,f1(x1)=%g,length=%g\n',x0,x1,f_0,f_1,len);
while((len > epsilon) && (k<iter))
    x0=x1;
    x1=x0-f1(x0)/f2(x0);
    f_0=f1(x0);
    f_1=f1(x1);
    len = x0-x1;
```

[2]https://github.com/chao92/Engineering-Optimization

```
    k=k+1;
    fprintf(1,'interation: %d\n',k);
    fprintf(1,'x0=%g,x1=%g,f1(x0)=%g,f1(x1)=%g,length=%g\n',x0,x1,f_0,f_1,len);
end
fprintf(1,'interation: %d\n',k);
fprintf(1,'x0=%g,x1=%g,f1(x0)=%g,f1(x1)=%g,length=%g\n',x0,x1,f_0,f_1,len);
```

Bisection Search:

```
% mailto:chao@ou.edu
% Bisection Search for minimization of function f(x)
% the same as find the root of f'(x)=0 assuming f'(a)<0 and f'(b)>0;
a = 0;                          % start of interval
b = 2.75;                       % end of interval
epsilon = 0.0001;               % accuracy value
k = 1;                          % number of iterations
iter = 500;                     % maximun number of iterations

len=b-a;
xm = (a + b)/2;
fm=f1(xm);
fa=f1(a);
fb=f1(b);
fxm=f(xm);
errors(k)=abs(fxm-f(1.5907));
fprintf(1,'interation: %d,error=%g\n',k,abs(fxm-f(1.5907)));
fprintf(1,'interval:(%g,%g),xm=%g,f1(x1)=%g,f1(xm)=%g,f1(x2)=%g,length=%g\n',a,b,xm,fa,fm,fb,len);
while((abs(fm) > epsilon) && (k<iter))
    if(fm > 0)
        b = xm;
    else
        a = xm;
    end
    k=k+1;
    len=b-a;
    xm = (a + b)/2;
```

```
        fm=f1(xm);
        fa=f1(a);
        fb=f1(b);
        fxm=f(xm);
        errors(k)=abs(fxm-f(1.5907));
        fprintf(1,'interation: %d,error=%g\n',k,abs(fxm-f(1.5907)));
        fprintf(1,'interval:(%g,%g),xm=%g,f1(x1)=%g,f1(xm)=%g,f1(x2)=%g,length=%g\n',a,b,xm,fa,fm,fb,len);
end
fprintf(1,'interation: %d,error=%g\n',k,abs(fxm-f(1.5907)));
fprintf(1,'interval:(%g,%g),xm=%g,f1(x1)=%g,f1(xm)=%g,f1(x2)=%g,length=%g\n',a,b,xm,fa,fm,fb,len);
```

Secant Search:

```
% mailto:chao@ou.edu
% Newton's search for minimization of function f(x)
% the same as find the root of f'(x)=0 assuming f'(a)<0 and f'(b)>0;
% f(x)=(x-2)^4+2(x-3)^2+3 -> f'(x)=4(x-2)^3+4(x-3) -> f''(x)=12(x-2)^2+4
epsilon = 0.0001;                                    % accuracy value
k = 0;                                               % number of iterations
iter = 500;                                          % maximun number of iterations

L=0;
R=2.75;

x1=R-f1(R)/((f1(R)-f1(L))/(R-L));
len = R-x1;
k=k+1;
errors(k) = abs(f(x1)-f(1.5907));
fprintf(1,'interation: %d, error=%g\n',k,abs(f(x1)-f(1.5907)));
fprintf(1,'L=%g,x1=%g,R=%g,length=%g\n',L,x1,R,len);
while((abs(f1(x1)) > epsilon) && (k<iter))

    if(f1(x1)>0)
        R=x1;
    else
        L=x1;
```

```
        end
        x1=R-f1(R)/((f1(R)-f1(L))/(R-L));
        len = R-x1;
        k=k+1;
        errors(k) = abs(f(x1)-f(1.5907));
        fprintf(1,'interation: %d, error=%g\n',k,abs(f(x1)-f(1.5907)));
        fprintf(1,'L=%g,x1=%g,R=%g,length=%g\n',L,x1,R,len);
end
fprintf(1,'interation: %d, error=%g\n',k,abs(f(x1)-f(1.5907)));
fprintf(1,'L=%g,x1=%g,R=%g,length=%g\n',L,x1,R,len);
```