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ISE 5643 — Spring 2015 **HW 1** Due: Feb. 2

Problem 1

Broker Steve Johnson is currently trying to maximize his profit in the bond market. Four bonds are available for purchase and sale, with the bid and ask price of each bond as shown in Table 1. Steve can buy up to 1000 units of each bond at the ask price or sell up to 1000 units of each bond at the bid price. During each of the next three years, the person who sells a bond will pay the owner of the bond the cash payments show in Table 2. Steve's goal is to maximize his revenue from selling bonds less his

Table 1

	BID PRICE	ASK PRICE
Bond 1	980	990
Bond 2	970	985
Bond 3	960	972
Bond 4	940	954

payment for buying bonds, subject to the constraint that after each year's payments are received, his current cash position (due only to cash payments from bonds and not purchases or sale of bonds) is nonnegative. Assume that cash payments are discounted, with a payment of \$1 one year from now being equivalent to a payment of \$.90 now. Formulate an LP to maximize net profit from buying and selling bonds, subject to the arbitrage constraints previously described. Why do you think we limit the number of units of each bond that can be bought or sold?

Table 2

YEAR	BOND 1	BOND 2	BOND 3	BOND 4
1	100	80	70	60
2	110	90	80	50
3	1100	1120	1090	1110

a. Define Variables
   B_i $i \in 1, 2, 3, 4$; denoted the bought of each kind of four bonds.
   S_j $j \in 1, 2, 3, 4$; denoted the sold of each kind of four bonds.
   C_k $k \in 0, 1, 2, 3$; denoted the cash at the end of each year.

b. Formulate Objective Function
   Maximize C_3.

c. Formulate Constraints
   s.t.
   $0 \leq B_1, B_2, B_3, B_4, S_1, S_2, S_3, S_4 \leq 1000$

$0 \le C_0, C_1, C_2$

$$\begin{aligned} First &: 980S_1 + 970S_2 + 960S_3 + 940S_4 - 990B_1 - 985B_2 - 972B_3 - 954B_4 = C_0 \\ Year1 &: -100S_1 - 80S_2 - 70S_3 - 60S_4 + 100B_1 + 80B_2 + 70B_3 + 60B_4 + C_0 = C_1 \\ Year2 &: -110S_1 - 90S_2 - 80S_3 - 50S_4 + 110B_1 + 90B_2 + 80B_3 + 50B_4 + C_1 = C2 \\ Year3 &: -1100S_1 - 1120S_2 - 1090S_3 - 1110S_4 + 1100B_1 + 1120B_2 + 1090B_3 + 1110B_4 \\ &+ C_2 = C_3 \end{aligned}$$

Considering the discounted:

$$\begin{aligned} Year1 &: C_1 = C_1/\frac{10}{9} \\ Year2 &: C_2 = C_2/(\frac{10}{9})^2 \\ Year3 &: C_3 = C_3/(\frac{10}{9})^3 \end{aligned}$$

d. Why do you think we limit the number of units of each bond that can be bought or sold?
Because if we do not have this limitation, then we cannot get our optimization solution.

Problem 2

Suppose a company must service customers lying in an area of **A** sq mi with n warehouses. Kolesar and Blum have shown that the average distance between a warehouse and a customer is

$$\sqrt{\frac{\mathbf{A}}{n}}$$

Assume that it costs the company \$60,000 per year to maintain a warehouse and \$400,000 to build a warehouse. (Assume that a \$400,000 cost is equivalent to forever incurring a cost of \$40,000 per year). The company fills 160,000 orders per year, and the shipping cost per order is \$1 per mile. If the company serves an area of 100 sq mi, how many warehouses should it have?

a. Define Variables
n number of warehouse.

b. Formulate Objective Function

$$f(n) = (1 \times \sqrt{\frac{100}{n}} \times 160,000) + (40,000 + 60,000) \times n$$

c. Formulate Constraints
s.t.
$n > 0$ *and* $n \in N$.

d. Opimal Solution

$$f^{'}(n) = 100,000 - 800,000 \times n^{-1.5} = 0$$

then, Get that, when n=4 the total cost is least.

Problem 3

The Wilcox Manufacturing Company produces an item that is subject to wide seasonal fluctuations in sale. The forecasted demand for the next 6 months for the forthcoming year are given below: The firm must always meet the monthly demand requirements.

Jan:	3000	Feb:	5000	March:	8000
April:	9000	May:	4000	June:	2000

It can fulfill the demand either by producing the required amount during the particular month or by producing only part of the required amount and making up the difference by using the overproduction (inventory) from previous months.

The Accounting Department has estimated that it costs \$2.00 to increase output by one unit from one month to the next and \$1.00 to reduce output by one unit from one month to another. The monthly cost of storing 1 unit of product is \$1.50. The warehouse capacity limits maximum inventory to 5000 units. The production capacity of Wilcox for any month is limited to 7000 units. The production schedule for the month of December in the current year calls for the production of 3000 units and it is estimated that the inventory level on January 1 will be 1000 units.

The problem is to determine the optimal production schedule for the Wilcox Company for the forthcoming year. Formulate this as an optimal control problem as follows:

a. Define your variables clearly. Identify the state variables and the control variables.
$i \in 1, 2, 3, 4, 5, 6$
P_i denoted the production of each month.
I_i denoted the inventory of each month.
N_i denoted the requirement of each month.
$P_0 = 3000$ and $I_0 = 1000$ denoted the initial production and inventory.
C denoted the cost for production and inventory.

b. Write down the constraints that must be satisfied each month briefly describing the significance of each.

- $P_i \leq 7000$ and $I_i \leq 5000$
- $P_i + I_{i-1} \geq N_i$ *where* $I_i = I_{i-1} + P_i - N_i$ $i \in [1, 6]$ where the production and inventory of last month must satisfy the need of current month, also inventory of current month is the production plus the last month's inventory minus the sell(market requirement).

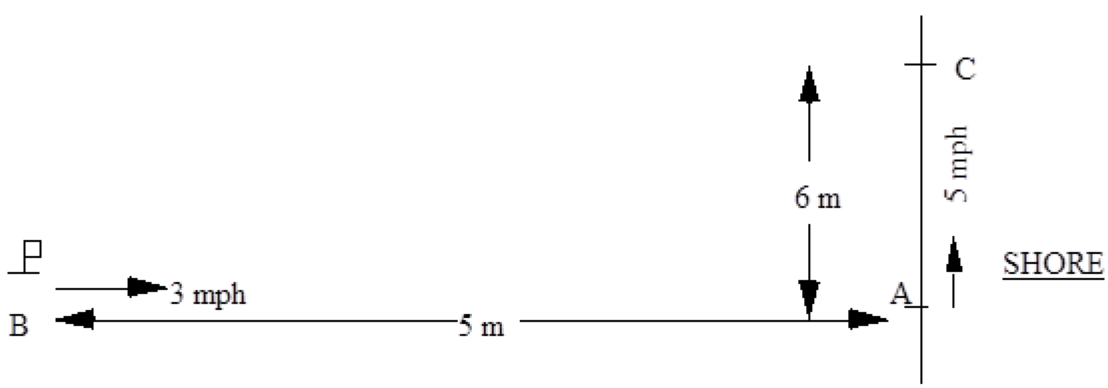
c. Write down the objective function to be optimized.
From the demand table, we know that the peak is on April, so we need increase our production from Jan to April, and decrease our production from May to June. So the objective function:

$$\begin{aligned} C &= 2*(P_1 - P_0) + I_0 * 1.5 + 2*(P_2 - P_1) + I_1 * 1.5 + 2*(P_3 - P_2) + I_2 * 1.5 \\ &+ 2*(P_4 - P_3) + I_3 * 1.5 + 1*(P_4 - P_5) + I_4 * 1.5 + 1*(P_5 - P_6) + I_5 * 1.5 \\ &= 7.5P_1 + 6P_2 + 4.5P_3 + 6P_4 + 1.5P_5 - P_6 - 123000 \end{aligned}$$

d. Show that the optimal control problem can be reduced to a linear programming problem.
We have six parameters need to be decided, also we have six constrain equations, so we can solve it as a linear programming problem.

Problem 4

A Beach Comber is 5 miles from a stretch of shore, rowing at 3 miles per hour (see figure below).



He can walk on the shore at a rate of 5 miles per hour. He wishes to reach a point (C) that is 6 miles downshore from the nearest point (A). He could row directly (from point B) to the desired destination (C), or he could row directly to the shore (point A) and walk to the desired destination (C), or he could do some combination of these. What route would get him to point C in the shortest time? Formulate this as a mathematical program.

<u>DO NOT SOLVE</u>. (Hint: Recall the Pythagoras theorem:)
Give a right angle triangle

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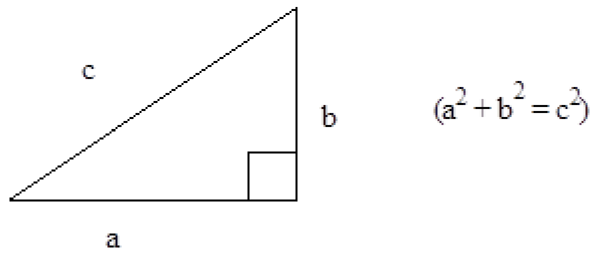

a. Define Variables
   x a random point D between B and A, the x denoted the distance between D and A. y a random point E between C and A, the y denoted the distance between E and A.

b. Formulate Objective Function
   Suppose his route is B $\longrightarrow D \longrightarrow E \longrightarrow C$, then we get the time from B to C is

$$f(x, y) = \frac{5 - x}{3} + \frac{\sqrt{x^2 + y^2}}{3} + \frac{6 - y}{5}$$

c. Formulate Constraints
   s.t.
   $x \in [0, 5]$
   $y \in [0, 6]$

d. Opimal Solution
   We need to find the x, y that minimize the objective function. By using Lingo, I got the answer is about 2.53.